On Decentralized Control of a Multimachine Power System: A Comparative study

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Abstract. In this paper, our focus is to compare two decentralized control laws implemented to a three-machine power system which generators are strongly nonlinear interconnected. We present first a linear decentralized control, which gains depend on the system nonlinearity, and confirm, via simulations, its ability to enhance the system transient stability. The second technique is a nonlinear decentralized optimal control based on a successive approximation approach where the nonlinear decentralized controllers are determined by the transformation of each high order coupling nonlinear two-point boundary value (TPBV) problem into a sequence of linear decoupling TPBV problems. We develop algorithmically and implement this decentralized optimal control to the three-machine power system. We prove, via advanced simulations, that this approach brings better performances than the linear decentralized controller, improving effectively transient stability of these power systems in a few steps.

Keywords. Power system, Decentralized control, Successive approximation approach.

1 Introduction

Power systems are classified as large-scale, distributed and highly non linear systems. They present generally fast transients which have been recently focused by many researches in order to improve the overall system transient stability ([1]-[3]). Centralized controllers are obviously not adapted to control such distributed systems because the global information of the entire system is not fully available in a centralized way to allow coordinating the control activity of the overall system. Furthermore, centralized controllers are technically and economically very difficult to design and implement for power systems modeled as complex and large-scale systems. As a logical alternative, decentralized control schemes are proposed, especially since power systems can be characterized by an interconnection of many subsystems. In fact, global central controllers can be substituted by local (decentralized) controllers designed especially for each sub-system. The main goal of decentralized control is to find some feedback laws to adapt the interactions from the other subsystems where no state information is transferred.
The advantage of this aspect in controller design is to reduce complexity, and minimize the amount of information transmission, which leads to better feasibility for the control implementation. Unlike centralized control, decentralized control cannot have access to the entire state information. Therefore, interconnections between subsystems need to be analyzed, so that their influence on the system performance can be properly addressed by the control. It is to be noted that the research on decentralized control has been prolific, founding applications in large space structures, power systems, communication networks, etc. ([8]-[16]-[26]-[27]-[28]). A wide variety of properties of the decentralized control systems are extensively studied in the literature and different design techniques are proposed accordingly ([17]-[20]).

In addition to the decentralized control, it is useful to reduce computation through modeling and decomposition techniques for complex systems. However, the simplified computation brings often a conservative result. To find a simpler analysis method or control strategy, we are always in front of high order, coupling and nonlinear TPBV problems which are generally impossible to resolve analytically [21]. This has inspired researches to look for some approaches to approximately obtain the solution to the nonlinear TPBV problem as well as obtain a suboptimal feedback control for nonlinear interconnected large-scale dynamic systems([24]-[25]).

Tang and Sun proposed in [22] a method of driving an optimal control for nonlinear interconnected large scale systems using a successive approximation approach, with respect to quadratic performance indexes. This technique transforms a high order coupling nonlinear TPBV problem into a sequence of linear decoupling TPBV problems.

In this paper, we are intended to use this result in order to develop algorithmically and implement a nonlinear decentralized optimal control for a nonlinear multimachine power system. A suboptimal control law can be obtained by using a finite iterative result of the optimal control sequence [22]. This technique is then compared with a linear decentralized control for the same multimachine power system. The paper is organized as follows: first we introduce in section II the description of the studied systems. Then, in section III, we present the decentralized linear control technique for nonlinear interconnected systems. Section IV focuses on the development of the algorithm for the optimal non linear decentralized control. In Section V, numerical simulations on a power system with three interconnected machines and a comparison study are given to highlight the efficiency of the proposed approaches. Finally, some conclusions are provided in section VI.

2 Description of the studied system

A nonlinear interconnected large-scale system decomposed into \( N \) subsystems, can be presented as follows:

\[
\begin{align*}
\dot{x}_i(t) &= A_i x_i(t) + B_i u_i(t) + f_i(t, x(t)), & t > t_0 \\
x_i(t_0) &= x_{i0}, & i = 1, 2, \ldots, N
\end{align*}
\]
where:
- \( x_i(t) \in \mathbb{R}^{n_i} \) is the state vector of the \( i^{th} \) subsystem,
- \( u_i \in \mathbb{R}^{m_i} \) is the control vector of the \( i^{th} \) subsystem,
- \( A_i \) and \( B_i \) are respectively the state matrix and the input matrix of each subsystem; \((A_i, B_i)\) are assumed to be controllable,
- \( x = [x_1, x_2, \ldots, x_N]^T \),
- \( n_1 + n_2 + \ldots + n_N = N \)
- \( f_i : C^1(\mathbb{R}^n) \to U_i \subseteq \mathbb{R}^{m_i} \), and \( f_i(t, x(t)) = f_i(x) \) expresses the interconnection terms vectors characterizing the nonlinearity of the \( i^{th} \) subsystem.

Assume that the nonlinear terms \( f_i(x) \) satisfy the Lipschitz conditions described by:

\[
\|f_i(x)\| \leq c\|x\|, \|f_i(x) - f_i(y)\| \leq h\|x - y\|, \forall x, y \in \mathbb{R}^n
\]  

where \( \|\cdot\| \) is the norm of vectors, \( c \) and \( h \) are known positive constants.

The control law \( u_i \) of each subsystem is a decentralized control. That is, it depends only on the information provided by the \( i^{th} \) subsystem, and does not need information exchange with the other subsystems.

We present in the following two approaches to calculate decentralized control for the studied interconnected non linear systems.

3 Decentralized linear control for the nonlinear interconnected systems

This section aims mainly to find a decentralized control for nonlinear interconnected system model described in (1) and verifying constraint (2). We suppose that the term of interconnection \( f_i(x) \) can be expressed as follows:

\[
f_i(x) = \sum_{j=1,j\neq i}^{N} G_{ij} g_{ij}(x_i, x_j), i = 1, \ldots, N
\]  

(3)

where \( G_{ij} \) is a matrix of appropriate dimension and \( g_{ij}(x_i, x_j) \) is a nonlinear function, satisfying the following inequality:

\[
\|g_{ij}(x_i, x_j)\| \leq \|W_i x_i(t)\| + \|W_{ij} x_j(t)\|
\]  

(4)

for all \( x_i \in \mathbb{R}^{n_i} \) and \( x_j \in \mathbb{R}^{n_j} \), where \( W_i \) and \( W_{ij} \) are two known and constant matrices.

The decentralized control is in a linear form; calculating control gains depends on the nonlinearity term of the power system.

In [23] the following theorem is demonstrated:

**Theorem 1** For each \( i = 1, 2, \ldots, N \), if there are \( R_i > 0 \) and defined positive matrix \( Q_i(n_i \times n_i) \) such that there is a positive defined matrix \( P_i(n_i \times n_i) \) which
4 Decentralized suboptimal control for interconnected systems

We consider the interconnected nonlinear system described by (1) and verifying constraint (2). The goal is to find an optimal control law that minimizes the quadratic cost function:

\[
J_i = \frac{1}{2} \sum_{i=1}^{N} \left( x_i^T(t_f)F_i x_i(t_f) + \int_{t_0}^{t_f} (x_i^T Q_i x_i + u_i^T R_i u_i) dt \right)
\]

(7)

Where \( Q_i \) and \( R_i \) are respectively the state and input weighting matrices that satisfy the general conditions of a linear-quadratic regulator.

If we apply the maximum principle to (1) and (7), then we can deduce that the necessary condition of the optimal control problem can be described as follows:

\[
\begin{aligned}
\dot{x}_i(t) &= A_i x_i(t) - S_i \lambda_i(t) + f_i(x), \quad t_0 < t < t_f \\
-\dot{\lambda}_i(t) &= Q_i x_i(t) + A_i^T \lambda_i(t) + \sum_{j=1}^{N} \sigma_{ij} \lambda_j(t), \quad t_0 \leq t < t_f \\
x_i(t_0) &= x_{i0}, \quad \lambda_i(t_f) = F_i x_i(t_f), \quad i = 1, 2, \ldots, N
\end{aligned}
\]

(8)

where \( S_i = B_i R_i^{-1} B_i^T \), \( \sigma_{ij} = \frac{\partial f_j}{\partial x_i} \) and \( \lambda_i \) is the adjoint vector introduced in the system Hamiltonian.

The resolution of the differential system (8) leads to the following decentralized optimal control law:

\[
u_i^*(t) = -R_i^{-1} B_i^T \lambda_i(t); \quad t_0 < t < t_f; \quad i = 1, 2, \ldots, N
\]

(9)

The nonlinear interconnected \( n^{th} \) order large-scale TPBV problems described by (8) can be decomposed into \( N \) sub-problems, which are very difficult to solve for general nonlinear interconnected function vectors \( f_i \). The goal is to develop some approximate approaches for solving the nonlinear large-scale interconnected TPBV problems in (8). Let’s consider now the following sequence
describing the linear TPBV problems:

\[
\begin{align*}
\dot{x}_i^{(k)}(t) &= A_i x_i^{(k)}(t) - S_i \lambda_i^{(k)}(t) + f_i(x^{(k-1)}), \quad t_0 < t \leq t_f \\
-\dot{\lambda}_i^{(k)}(t) &= Q_i x_i^{(k)}(t) + A_i^T \lambda_i^{(k)}(t) + \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t), \quad t_0 < t < t_f \\
x_i^{(k)}(t_0) &= x_{i0}, \quad \lambda_i^{(k)}(t_f) = F_i x_i(t_f), \\
i &= 1, 2, \ldots, N, k = 1, 2, \ldots
\end{align*}
\]

(10)

where \( f_i(x^{(0)}) = 0 \), \( \lambda_i^{(0)}(t) = 0 \), \( \sigma_{ij}^{(k-1)} \) are known. Therefore, (10) is a linear nonhomogeneous TPBV problem.

Let:

\[
\lambda_i^{(k)}(t) = P_i(t) x_i^{(k)}(t) + g_i^{(k)}(t), \quad i = 1, 2, \ldots, N
\]

(12)

Where \( P_i(t) \) is the unique semi-positive definite matrix of the following Riccati matrix differential equation:

\[
\dot{P}_i(t) + P_i(t) A_i + A_i^T P_i(t) - P_i(t) S_i P_i(t) + Q_i = 0
\]

(13)

Substituting (12) into (10), we get a sequence of adjoint vector differential equations:

\[
\begin{align*}
\dot{g}_i^{(k)}(t) &= (P_i(t) S_i - A_i^T P_i(t) f_i(x^{(k-1)})) - \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t), \quad t_0 \leq t < t_f \\
g_i^{(k)}(t_f) &= 0, \quad i = 1, 2, \ldots, N, k = 1, 2, \ldots
\end{align*}
\]

(14)

From development above, we can deduce the model for the studied system as follows:

\[
\begin{align*}
\dot{x}_i^{(k)}(t) &= A_i x_i^{(k)}(t) - S_i \lambda_i^{(k)}(t) + f_i(x^{(k-1)}), \\
\dot{g}_i^{(k)}(t) &= (P_i(t) S_i - A_i^T P_i(t) f_i(x^{(k-1)})) - \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t) \\
-\dot{\lambda}_i^{(k)}(t) &= Q_i x_i^{(k)}(t) + A_i^T \lambda_i^{(k)}(t) + \sum_{j=1}^{N} \sigma_{ij}^{(k-1)} \lambda_j^{(k-1)}(t) \\
x_i^{(k)}(t_0) &= x_{i0}, \quad \lambda_i^{(k)}(t_f) = F_i x_i(t_f), \quad g_i^{(k)}(t_f) = 0 \\
t_0 \leq t < t_f, i = 1, 2, \ldots, N, k = 1, 2, \ldots
\end{align*}
\]

(15)

The sequence solution to TPBV problem sequence (10) uniformly converges to the solution of large-scale nonlinear interconnected TPBV problem, and TPBV
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problem sequence (15) becomes solvable and uniformly convergent [22]. According to (11), \( u_i^{(k)}(t) \) is also uniformly converging to optimal control \( u_i^*(t) \). Therefore:

\[
\begin{align*}
    u_i^*(t) &= \lim_{k \to \infty} u_i^{(k)}(t) = -R_i^{-1}B_i^T[P_i(t)x_i(t) + g_i^{(k)}(t)] \\
    t_0 \leq t &\leq t_f, i = 1, 2, \ldots, N
\end{align*}
\] (16)

Replacing \( k \to \infty \) with \( k = M \) in (16), we may obtain a suboptimal control law useful in practical application. And we consider the \( \lambda_i^* \) for large scale system (21) is obtained as follows:

\[
\begin{align*}
    u_{iM}(t) &= -R_i^{-1}B_i^T[P_i(t)x_i(t) + g_i^{(M)}(t)], \\
    t_0 \leq t &\leq t_f, i = 1, 2, \ldots, N
\end{align*}
\] (17)

In practical control systems, we may consider \( t_f \to \infty \) when \( t_f \) is large enough. Therefore, this approach is also applicable for the case of \( t_f \to \infty \). Cost functional (7) becomes:

\[
J_i = \frac{1}{2} \sum_{r=1}^{N} \int_{t_0}^{t_f} (x_i^TQ_ix_i + u_i^TR_iu_i)dt
\] (18)

Therefore, the following algebraic Riccati matrix equation is used instead of the algebraic Riccati equations in (25). We give a positive constant \( \epsilon \). Let \( k = 1, M = 1 \) and \( x_i^{(0)}(t) = g_i^{(0)}(t) = g_i^{(1)}(t) = 0 \)

- **Step 1**: We extract the semi-positive definite matrices \( P_i(t) \) from the algebraic Riccati equations in (25). We give a positive constant \( \epsilon \). Let \( k = 1, M = 1 \) and \( x_i^{(0)}(t) = g_i^{(0)}(t) = g_i^{(1)}(t) = 0 \)

- **Step 2**: We obtain \( x_i^{(1)}(t) \) from:

\[
\begin{align*}
    \dot{x}_i^{(1)} &= (A_i - S_iP_i(t))x_i^{(1)}(t) & t_0 \leq t < t_f \\
    x_i^{(1)}(t_0) &= x_{i0}, & i = 1, 2, \ldots, N
\end{align*}
\] (20)

and this gives \( \lambda_i^{(1)}, f_i(x^{(1)}) \) and \( \sigma_{ij}^{(1)} \). We get \( u_{i1}(t) \) from (17) and \( J_1 \) from (7). Let \( k = k + 1 \).

- **Step 3**: Letting \( M = k \), we find \( g_i^{(k)} \) from (15). \( u_{iM}(t) \) is therefore obtained from (17). We calculate \( J_M \) according to (7)

- **Step 4**: If \( \frac{|J_M - J_{M-1}|}{J_M} < \epsilon \), then we stop and extract the suboptimal control law \( u_{iM}(t) \).

- **Step 5**: We deduce \( f_i(x^{(k)}) \) from (15). This will lead to find \( \lambda_i^{(k)}, f_i(x^{(k)}) \)

\[ \text{and } \sigma_{ij}^{(k)} \]. Letting \( k = k + 1 \), we go to step 3.
5 Application of the proposed decentralized approaches to an interconnected multimachine power system

The purpose of this section is to implement the decentralized control methods presented in the previous paragraphs, on a power system with three interconnected machines (Fig. 1), in order to analyze the performances due to the application of each approach.

5.1 Multimachine Power System nonlinear Model

A large scale power system $S$ with steam valve control can be described by the interconnection of $N$ subsystems (or synchronous generators) $S_i$, $i = 1, \cdots, N$. The mathematical model of the subsystems is described by the following equations [23]:

$$
\dot{x}(t) = A_i x_i(t) + B_i u_i(t) + \sum_{j=1, j \neq i}^{N} p_{ij} G_{ij}(x_i, x_j), i = 1, \cdots, N \quad (21)
$$

where $x_i(t)$ is the state vector defined by:

$$
x_i(t) = [\Delta \delta_i(t), \omega_i(t), \Delta P_{m_i}(t), \Delta X_{e_i}(t)]^T,
$$

with:

- $p_{ij}$ a constant of either 1 or 0 (=0 means that $S_i$ has no connection with $S_j$);
- $\delta_i$ the rotor angle for $S_i$, in radian;
- $\omega_i$ the relative speed for $S_i$, in radian/s;
- $P_{m_i}$ the mechanical power for $S_i$, in pu;
- $X_{e_i}$ the steam valve opening for $S_i$, in pu;
- $H_i$ the inertia constant for $S_i$, in second;
- $D_i$ the damping coefficient for $S_i$, in pu;
- $T_{m_i}$ the time constant for $S_i$’s turbine, in second;
- $K_{m_i}$ the gain of $S_i$’s turbine;
- $T_{c_i}$ the time constant of $S_i$’s speed governor, in second;
- $K_{c_i}$ the gain of $S_i$’s speed governor;
- $R_i$ the regulation constant of $S_i$, in pu;
- $B_{ij}$ the nodal susceptance between $S_i$ and $S_j$, in pu;
- $\omega_0$ the synchronous machine speed, in radian/s;
- $E_{q_i}$ the internal transient voltage for $S_i$, in pu, which is assumed to be constant;
- $E_{q_j}$ the internal transient voltage for $S_j$, in pu, which is assumed to be constant;

and:

$$
A_i = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -D_i & -\frac{\omega_0}{2H_i} & 0 \\
0 & 0 & -\frac{K_{m_i}}{T_{m_i}} & \frac{1}{T_{c_i}} \\
0 & -\frac{K_{c_i}}{T_{c_i}} & 0 & -\frac{1}{T_{c_i}}
\end{bmatrix},
$$

$$
B_i = \begin{bmatrix}
0 \\
0 \\
0 \\
0
\end{bmatrix},
$$

$$
G_{ij} = \begin{bmatrix}
0 & 0 \\
-\frac{\omega_0 E_{q_i} E_{q_j} B_{ij}}{2H_i} \\
0 \\
0
\end{bmatrix},
$$

g_{ij}(x_i, x_j) = \sin (\delta_i(t) - \delta_j(t)) - \sin (\delta_0 - \delta_j(t))
\( \delta_i, P_m, \) and \( X_e \) are the initial values of \( \delta_i(t), P_m(t) \) and \( X_e(t) \) respectively. The parameters of the power system with three interconnected machines are summarized in Table 1 [23].

<table>
<thead>
<tr>
<th>Machine Parameters</th>
<th>Machine 1</th>
<th>Machine 2</th>
<th>Machine 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_d (pu) )</td>
<td>1.863</td>
<td>2.36</td>
<td>2.36</td>
</tr>
<tr>
<td>( x'_d (pu) )</td>
<td>0.257</td>
<td>0.319</td>
<td>0.319</td>
</tr>
<tr>
<td>( x_T (pu) )</td>
<td>0.129</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>( x_{od} (pu) )</td>
<td>1.712</td>
<td>0.712</td>
<td>0.712</td>
</tr>
<tr>
<td>( T_{d0} (pu) )</td>
<td>6.9</td>
<td>7.96</td>
<td>7.96</td>
</tr>
<tr>
<td>( H(s) )</td>
<td>4</td>
<td>5.1</td>
<td>5.1</td>
</tr>
<tr>
<td>( D(pu) )</td>
<td>5</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>( T_m(s) )</td>
<td>0.35</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>( T_e(s) )</td>
<td>0.1</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( R )</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>( K_m )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>( K_e )</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

According to model (21), the studied power system can be described by the
following state equations:

\[
\begin{align*}
\dot{x}_1(t) &= A_1 x_1(t) + B_1 u_1(t) + G_{12} g_{12}(x_1, x_2) + G_{13} g_{13}(x_1, x_3) \\
\dot{x}_2(t) &= A_2 x_2(t) + B_2 u_2(t) + G_{21} g_{21}(x_2, x_1) + G_{23} g_{23}(x_2, x_3) \\
\dot{x}_3(t) &= A_3 x_3(t) + B_3 u_3(t) + G_{31} g_{31}(x_3, x_1) + G_{32} g_{32}(x_3, x_2)
\end{align*}
\]

where:

- \( G_{ij} = \begin{bmatrix} 0 & \alpha_{ij} & 0 \\ \alpha_{ij} & 0 & 0 \end{bmatrix}^T \)
- \( \alpha_{ij} \) represents the midpoints of \( \frac{\omega_0 E'_q E'_q B_{ij}}{2H_i} \)

In this case, the values of \( \alpha_{ij} \) are taken as follows [23]:

\( \alpha_{12} = \alpha_{13} = -27.49, \alpha_{21} = \alpha_{23} = \alpha_{31} = \alpha_{32} = -23.10 \)

### 5.2 Application of the decentralized linear control

The decentralized linear control law (6), applied to the three-machine power system, is defined by:

\[
u_i(t) = -K_i x_i(t) = -K_{\delta_i} [\delta_i(t) - \delta_{i0}] - K_{\omega_i} (t) - K_{P_i} [P_{m_i}(t) - P_{m_i0}] - K_{X_i} [X_{e_i}(t) - X_{e_i0}]
\]

where:

\[ K_i = R_i^{-1} B_i^T P_i \]

with \( P_i \) the symmetric positive solution of the modified Riccati equation (5) for \( i = 1, 2, 3 \). The weighting matrices \( R_i = 2 \) and \( Q_i = \text{diag}(0.001, 0.001, 0, 01, 0.01) \), \( i = 1, 2, 3 \);

Based on the parameters above, we can derive the following decentralized control gains:

\[
K_1 = \begin{bmatrix} -1.11 & -1.16 & -9.03 & -3.32 \end{bmatrix} \\
K_2 = \begin{bmatrix} -55.8 & -33 & -159.28 & -106.62 \end{bmatrix} \\
K_3 = \begin{bmatrix} -55.8 & -33 & -159.28 & -106.62 \end{bmatrix}
\]

Our goal is to prove the performance of the proposed linear decentralized controller for the following operation points of the power system variables: \( \delta_{i0} = 24.6rad, P_{m_i0} = 1pu, X_{e_i0} = 1pu \).

Figure 2 shows the state variable evolution (of the controlled system) towards a perturbation on the rotor angle of the second generator. Figure 3 illustrates the corresponding control signal evolution. These simulations confirm the ability of the proposed linear decentralized control to enhance the system transient stability. But we can also notice clearly the limitation of this control: the transient regime is highly oscillatory and cannot dampen out rapidly the oscillations generated by perturbations.
5.3 Application of the decentralized nonlinear control

We propose to be within the same conditions for simulation in subsection 5.1, and apply therefore the nonlinear decentralized control using the successive ap-
proximation approach to the three machine power system choosing the same perturbation in the rotor angle of second machine. The weighting matrices are $R_i = 2$ and $Q_i = \text{diag}\{0.001, 0.001, 0.01, 0.01\}$, $i = 1, 2, 3$.

First, we extract the semi-positive definite matrices $P_i(t)$, $P_2(t)$ and $P_3(t)$ from the algebraic Riccati matrix equations as follows:

$$P_i A_i + A_i^T P_i - P_i S_i P_i + Q_i = 0, \quad i = 1, 2, 3 \quad (25)$$

and let $k = 1$, $M = 1$ and $x_i^{(0)}(t) = g_i^{(0)}(t) = g_i^{(1)}(t) = 0$. Then we obtain $x_i^{(1)}$ from:

$$\begin{cases} \dot{x}_i^{(1)} = (A_i - S_i P_i(t)) x_i^{(1)}(t) & t_0 \leq t < t_f \\ x_i^{(1)}(t_0) = x_{i0}, & i = 1, 2, 3 \end{cases} \quad (26)$$

Then we get $u_i$ from (17) and $J_1$ from (7). The next step is to increment $k$ into $k + 1$ to obtain $g_i^{(2)}$ for $M = 2$ and calculate $u_{i2}(t)$.

After the third iteration of the control, we can deduce the cost functionals of composite system (22). $J_1 = 0.9934$, $J_2 = 0.967$ and $J_3 = 0.7712$. As expected: $J_1 > J_2 > J_3$. If $\epsilon = 0.15$, $|J_3 - J_2|/J_3 = 0.102 < \epsilon$. Therefore, the control precision can be obtained after 3 iterations. When $k = 1, 2, 3$ the simulation curves of state variable evolution and the corresponding control signals are shown in Figure 4, Figure 5, Figure 6, Figure 7, Figure 8 and Figure 9. It is clearly proven that within simulations advancement through iterations, the precision is getting progressively better. Simulation results presented in Figure 8 and Figure 9 demonstrate the ability of the decentralized nonlinear control to mitigate rapidly the effect of the occurred fault location, neutralizing the oscillations and improving rapidly transient stability of the multimachine power system even though it presents strong nonlinear interconnections between its generators.
6 Conclusion

This paper is interested in the implementation of two different decentralized controls to a 3-machine large-scale power system which generators are strongly
nonlinear interconnected. First we applied the conventional linear decentralized control law obtained by solving a modified algebraic Riccati equation that depends on the nonlinearity of the large-scale system. The second approach required to develop algorithmically and apply a nonlinear decentralized optimal control scheme which control laws are calculated using a successive approximation approach. A suboptimal control law has been obtained by using a finite iterative result of optimal control law sequence. Simulation results demonstrate that the nonlinear decentralized controller provides better performance, and out-
Fig. 9. Control signal evolution (k=3)

performs the optimized conventional linear decentralized control. In fact, the nonlinear decentralized controller based on successive approximation approach, proved more efficiency in damping out oscillations of the power system within few iterative sequences, improving significantly the system transient stability.
Bibliography


