Robust Decoupling Control for Mobile Manipulators Based on Disturbance Observer

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Abstract. This paper describes a robust approach exploiting input-output decoupling controller for mobile manipulators. The robust controller consists of using a classical proportional-derivative (PD) feedback structure to stabilize the position error plus an additional disturbance observer (DOB) to compensate external disturbances and uncertainties. Simulation results on a mobile platform with 2-DOF manipulator show the satisfactory performance of the proposed control schemes even in the presence of large modeling uncertainties and external disturbances.

keywords. Mobile manipulators, input-output decoupling controller, disturbance observer.

1. Introduction

A mobile manipulator refers to the mobile system that has a mobile platform carrying a robotic manipulator. Such systems combine the advantages of mobile platforms and robotic arms to reduce their drawbacks. For instance, the mobile platform extends the arm workspace, whereas an arm offers much operational functionality. Applications for such systems could be found in mining, construction, forestry, planetary exploration, and human assistance [1, 2, 3]. The combined system introduces new issues that are not present in the analysis of each subsystem considered separately. First, the dynamics of the combined system are much more complicated because they include dynamic interactions between mobile platform and manipulator. Second, due to complex structure of the mobile manipulator, the constraints which are valid only for one subsystem will also hold for the whole
mobile manipulator. Third, combining the mobility of the base platform and the manipulator creates redundancy since the combined system typically possesses more degrees of freedom than necessary. In particular, any given point in the workspace may be reached by the motion of the mobile platform only, by the motion of the onboard manipulator only or by the coordinated motion of both subsystems.

Although a control problem of mobile manipulators has attracted more attention over last decade, see for e.g. [4, 5, 6, 7], yet an approach exploiting input-output decoupling controller can be found only in works of few authors: Yamamoto and Yun [8-9] positioned the manipulator at the preferred configurations measured by its manipulability. The output functions to define an input-output decoupling controller are the coordinates of the end-effector, if the onboard manipulator stays in constant configuration relative to the mobile platform. It means that the desired task formulated for the whole mobile manipulator can be realized by the motion of the mobile platform only. Mazur [10-11] extended the output functions to the more general form, such a choice of the output functions makes possible to exploit both manoeuvres of the nonholonomic mobile platform and the motion of the onboard manipulator. Rajankumar [12] studied the effect of the output function on the controllability of a mobile platform. Most previous approaches require a precise knowledge of the kinematics and the dynamics of mobile manipulators and ignore disturbances and uncertainties, these issues make the proposed schemes inappropriate for realistic applications.

In this paper, an input-output decoupling robust control technique with a classical proportional derivative (PD) feedback structure is proposed to control the position of the mobile manipulator. The controller is followed by a disturbance observer (DOB) to compensate the dynamic effects including the bounded known/unknown disturbances and uncertainties.

This paper is organized as follows. Section 2 is devoted to mathematical description of the mobile manipulator with nonholonomic constraints. Section 3 presents the design of the input-output decoupling controller for nonholonomic mobile manipulator. In section 4 the proposed controller design procedure is formulated. Section 5 presents computer simulation results to illustrate the effectiveness of the proposed technique. Conclusions are formulated in Section 6.

2. Dynamic Model of a mobile Manipulator

In this section, we consider the mobile manipulator consisting of two subsystems, namely a nonholonomic wheeled mobile platform and holonomic rigid manipulator whose schematic top view is shown in figure 1.
The dynamics of a mobile manipulator subject to nonholonomic constraints can be obtained using the Lagrangian approach in the following form [13]:

\[ M(q)\ddot{q} + C(q, \dot{q}) + \lambda^T A(q) = E(q)\tau + F(q)d \]  

(1)

where \( q = (q_1^T, q_2^T)^T \in \mathbb{R}^{n_d} \) denote the \( n \) generalized coordinates, \( M(q) \in \mathbb{R}^{n_d \times n_d} \) is a symmetric and positive definite inertia matrix, \( C(q, \dot{q}) \in \mathbb{R}^{n_d} \) represents the vector of centripetal and Coriolis forces terms \( A(q) \in \mathbb{R}^{n_d} \) is the constraint matrix, \( \lambda \in \mathbb{R}^{n_d} \) is the vector of the Lagrange multipliers which denotes the vector of the constraint forces, \( E(q) \in \mathbb{R}^{(n-m) \times n_d} \) is the input transformation matrix, \( \tau \in \mathbb{R}^{(n-m)\times m} \) is the vector of input torques, \( F(q) \in \mathbb{R}^{n_d} \) denotes the disturbance gain matrix and \( d \in \mathbb{R}^n \) represents the vector of disturbance affecting the system.

In order to eliminate the constraint force \( \lambda \), let \( S(q) \) be an \( (n-m) \times (n-m) \) full rank matrix formed by a set of smooth and linearly independent vector fields spanning the null space of \( A(q) \), i.e., \( S(q)A(q) = 0 \); thus, constraint equations (2) implies the existence of a vector \( \eta \in \mathbb{R}^{(n-m)d} \), such that:

\[ \dot{\eta} = S(q)\eta \]  

(3)
The dynamic equation (1) can therefore be rewritten as:

$$\ddot{\mathbf{M}}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \ddot{\mathbf{E}}(\mathbf{q})\tau + \ddot{\mathbf{F}}(\mathbf{q})d$$  \hspace{1cm} (4)

where \( \ddot{\mathbf{M}}(\mathbf{q}) = \mathbf{S}^T \mathbf{M} \mathbf{S}, \mathbf{C}(\mathbf{q}, \dot{\mathbf{q}}) = \mathbf{S}^T (\mathbf{C} + \mathbf{M}\ddot{\mathbf{q}}), \mathbf{E}(\mathbf{q}) = \mathbf{S}^T \mathbf{E}, \mathbf{F}(\mathbf{q}) = \mathbf{S}^T \mathbf{F} \)

Using the state vector \( \mathbf{x} = \begin{bmatrix} \dot{\mathbf{q}}^T & \ddot{\mathbf{q}}^T \end{bmatrix}^T \), we allow us to represent the constraint and motion equations of the mobile manipulator in state space:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\tau$$  \hspace{1cm} (5)

where: \( \mathbf{f}(\mathbf{x}) = \begin{bmatrix} \mathbf{S}\ddot{\mathbf{q}}^T & \mathbf{M}^{-1}(\ddot{\mathbf{d}} - \ddot{\mathbf{C}})^T \end{bmatrix}^T \) and \( \mathbf{g}(\mathbf{x}) = \begin{bmatrix} 0 & \mathbf{M}^{-1}\ddot{\mathbf{E}}^T \end{bmatrix}^T \)

3. Problem Formulation

3.1. Necessary Condition for Existence of the Input-Output Decoupling Controller for Nonlinear Affine Control systems

It is well known [14], that a smooth affine nonlinear control system

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{m} g_i(\mathbf{x})u_i, \ \mathbf{x} \in \mathbb{R}^n$$  \hspace{1cm} (6)

and the output function

$$y = h(\mathbf{x}), \ y \in \mathbb{R}^p$$  \hspace{1cm} (7)

can be input-output decoupled if the following conditions hold

- \( p = m \)
- there exist finite nonnegative integers \( \rho_1, \ldots, \rho_m \) such that

$$L_{\mathbf{g}_i} L_{\mathbf{j}} h_i = 0, \ k = 0, 1, \ldots, \rho_i - 1,$$

$$L_{\mathbf{g}_i} L_{\mathbf{j}} h_i \neq 0, \ i = 1, \ldots, m$$  \hspace{1cm} (8)

The above conditions imply, that \((\rho_i + 1)\) time derivative of \(i\)th output can be controlled by the \(i\)th input, and has a form

$$y^{\rho_i+1}_i = L_{\mathbf{g}_j}^{\rho_i+1} h_i + L_{\mathbf{g}_i} L_{\mathbf{j}}^{\rho_i} h_i u_j, \ i = 1, \ldots, m,$$  \hspace{1cm} (9)

where \( L_{\mathbf{g}_i}, h_j \) and \( L_{\mathbf{j}} h_j \) are the Lie derivatives of a function \( h_j \) along smooth vector fields \( \mathbf{g}_i \) and \( \mathbf{j} \), respectively.
If we want to obtain input-output decoupled system, it is necessary to use the following control law,

\[ u_i = \alpha_i(x) + \beta_i(x) \zeta_i \]  

(10)

where \( \alpha_i(x) = \phi_i^{-1}(-L_i^{\delta+1} h_i) \), \( \beta_i(x) = \phi_i^{-1} \) and \( \phi_i = L_{e_i} L_{y_i} L_{h_i} \) is the decoupling matrix.

The next step is to apply the feedback law (10) to the system (9) such that each of the \( m \) outputs will be controlled by one of the new defined inputs \( \zeta_i \)

\[ y_i^{\delta+1} = \zeta_i, \quad i = 1, ..., m. \]  

(11)

### 3.2. Decoupling Controller for Mobile Manipulator

In this paper, the desired trajectory in the workspace can be realized by the coordinated motion between the mobile platform and the onboard manipulator. To avoid redundancy in design of the decoupling controller, we defined the output function for the mobile manipulator as follows:

\[ y = h(x) = \begin{pmatrix} y_1(q_r) \\ y_2(q_r) \end{pmatrix} \]  

(12)

Where \( y_1(q_r) \) describe position of the end-effector relative to inertial frame when the manipulator is in the constant optimal configuration that maximizes manipulability measure, and \( y_2(q_r) \) describe the position of the end-effector relative to this optimal configuration. Such a choice of the output function facilitates the coordination between the manipulator and the mobile platform.

We then design a decoupling controller that enables the end-effector to track a desired trajectory. For this purpose we differentiate the output function provides a reference for the equation:

\[ \dot{y}(x) = \begin{pmatrix} \dot{y}_1(q_r) \\ \dot{y}_2(q_r) \end{pmatrix} = \begin{bmatrix} \frac{\partial y_1}{\partial q_r} S(q_r) & 0 \\ 0 & \frac{\partial y_2}{\partial q_r} \end{bmatrix} \eta = \begin{bmatrix} \Phi_x & 0 \\ 0 & \Phi_r \end{bmatrix} \eta \]  

(13)

The first derivative of the output function

\[ \dot{y} = \Phi(x) \eta \]  

(14)

does not depend on the input signal. After the second differentiation we get:

\[ \ddot{y} = \Phi(x) \eta + \Phi(x) u \]  

(15)
We see that for the proper input-output decoupling and linearization, the following feedback control is needed, namely

\[ u = \Phi^{-1}(x)(\zeta - \Phi(x)\eta) \]  

(16)

where \( \zeta \) is a new input for the decoupled system.

The proposed control law can be realized if the matrix \( \Phi \) is invertible.

\[ \Phi^{-1}(x) = \begin{bmatrix} \Phi^{-1} & 0 \\ 0 & \Phi^{-1} \end{bmatrix} \]  

(17)

The matrix \( \Phi \) is invertible if the configuration of the manipulator is non-singular. For this reason we propose to choose the configuration that maximizes the manipulability of the manipulator which should be as far as possible from any singularity. Also, the starting position of the manipulator relative to its base has to be enough far from any singular configuration.

4. Design of the Proposed Controller

The proposed controller as shown in figure 2 is made up of two controllers that could be theoretically designed independently. A (PD) controller to preserve the trajectory tracking for the decoupled mobile manipulator and the disturbance observer controller (DOB) applied to the mobile manipulator in joint-space to accomplish the robust acceleration control system.

![Fig. 2. Block Diagram of the Proposed Controller.](image-url)
4.1. Proportional-Derivative (PD) Controller

The PD-controller exists in the outermost loop of the controller with correction as follows

$$\zeta = \ddot{y}_d - K_p(y_d - y) + K_d(\dot{y}_d - \dot{y})$$  \hspace{1cm} (18)

where: $y$, $\dot{y}$, and $\ddot{y}$ are respectively the actual position, velocity and acceleration of the mobile manipulator, $y_d$, $\dot{y}_d$, and $\ddot{y}_d$ are respectively the desired position, velocity and acceleration of the mobile manipulator, $K_p$ and $K_d$ are the proportional and derivative gains respectively. Such controller is sufficient to preserve the trajectory tracking for the decoupled mobile manipulator. The PD-control gains should have high values to preserve the convergence without any overshoot [10-11].

4.2. Disturbance Observer (DOB) Controller

Disturbance observer (DOB) is known to be an effective method to compensate for the disturbance of a motion control system [16-17-18]. Since (DOB) presents a simple structure and a low computation effort, it has been adopted in this paper in order to realize precise and robust control. In this approach the influence of disturbances can be estimated by a DOB inner loop and can be suppressed by adding the estimated disturbance signal to the control input. Its basic concept is shown in Fig.2 which is based on estimate the disturbance from the acceleration response of the system and the reference input.

To characterize the estimation process, the actual disturbance can be derived using the dynamics of the system (4) as:

$$d = \ddot{\bar{F}}(q)^{-1}(\dddot{M}(q)\ddot{q} + \dddot{C}(q, \dot{q}) - \dddot{E}(q)\tau)$$  \hspace{1cm} (19)

A non linear disturbance observer is proposed as:

$$\dot{\hat{d}} = -L(q)\hat{d} + L(q)\dddot{\bar{F}}(q)^{-1}(\dddot{M}(q)\ddot{q} + \dddot{C}(q, \dot{q}) - \dddot{E}(q)\tau)$$  \hspace{1cm} (20)

Since, in general, there is no prior information about the derivative of the disturbance $d$, it is reasonable to suppose that $\dot{d} = 0$.

Which imply that the disturbance varies slowly relative to the observer dynamics. However, it will be shown in [19] that the DOB controller can track some fast time varying disturbances when the parameters of the observer are properly chosen.

The observer error is defined as:

$$\epsilon(t) = d - \hat{d}$$  \hspace{1cm} (21)

By (19) and (21), we have

$$\dot{\epsilon}(t) = d - \hat{d} = L(q)\dot{\hat{d}} - L(q)d$$  \hspace{1cm} (22)
Hence by (21) and (22), the error dynamics of the observer are given by:

\[ \dot{e} + L(q)e = 0 \]  

(23)

It follows therefore from (23) that a nonlinear disturbance observer given in (20) is globally asymptotically stable, for all allowable \( q \), if and only if \( -L(q) \) is a Hurwitz matrix; that is, if and only if the real parts of all eigenvalues (real or complex) of \( -L(q) \) are strictly negative. In meeting this stability requirement, the matrix \( L(q) \) can assume many different forms. A particularly simple choice of \( L(q) \) for stability is

\[ L(q) = \text{diag}\{l_1, \ldots, l_i, \ldots, l_n\} \]  

(24)

where \( l_i > 0 \), and the exponential convergence rate of the observer error dynamics (23) can be specified by appropriate choice of \( l_i \). This implies that different convergence rates can be specified for different disturbances in the disturbance vector \( d \in \mathbb{R}^n \) in (4).

5. Simulations

The experiments were developed under the MATLAB simulation environment. The aim is to clearly demonstrate the advantages of the proposed scheme. The geometric and dynamic parameters for the mobile manipulator (Figure 1) are shown in Table 1.

Table 1. Parameters of the Mobile Manipulator

<table>
<thead>
<tr>
<th>Parameters (Units)</th>
<th>Platform</th>
<th>Manipulator</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension (mm)</td>
<td>b=168, d=100, r=51</td>
<td>l_1=508, l_2=508</td>
</tr>
<tr>
<td>Mass (kg)</td>
<td>m_c=17.25, m_w=0.1</td>
<td>m_1=1.07, m_2=1.07</td>
</tr>
<tr>
<td>Inertia (kg mm^2)</td>
<td>I_c=3.10^{-5}</td>
<td>I_1=2.6.10^{-4}, I_2=2.6.10^{-4}</td>
</tr>
</tbody>
</table>

The desired trajectory for the mobile manipulator has been chosen as follows:

\[ \begin{pmatrix} x_d(t) \\ y_d(t) \end{pmatrix} = \begin{pmatrix} 2\cos(t) + 3 \\ 2\sin(t) + 3 \end{pmatrix} \]  

(25)

The initial positions of the platform are set to: \((x(0), y(0), \varphi(0))=(6,4,\frac{\pi}{2})\) and the initial positions of the manipulator variables are set to \((\theta_1(0), \theta_2(0))=(-\frac{\pi}{4}, \frac{\pi}{4})\).

The initial experiment was conducted to determine the appropriate values of \( K_p \) and \( K_d \) of the PD-controller. The tuning process was performed in the PD-controller mode considering some disturbances in the process. By using the tuned \( K_p \) and \( K_d \)
values, a number of experiments were then performed including the disturbance observer scheme to approximate the value of the inertia matrix \( L \).

Two types of disturbances are applied to each wheel and joint of the mobile manipulator. The first is in the form of vibratory excitation (Figure 3) and the other is in the form of impact forces (Figure 6), it represents the conditions of the mobile robot encountering an instantaneous ‘collision’ with an obstacle along its path or is hitting a ‘bump’ or ‘hole’ while navigating.

From the simulation results, the gains for the PD-controller in the control algorithm are equal to \( K_p = 100 \) and \( K_d = 70 \), these values will be enough to obtain smooth trajectory tracking errors. These values would subsequently be used in the next investigation employing the disturbance observer scheme, to tune \( L \) which was obtained as: \( L = \text{diag}[0.02, \ 0.02, \ 0.1, \ 0.1] \).

In the presence of external disturbances, the performances of the feedback (PD) structure only and a PD plus an additional disturbance observer (PD-DOB) controller are presented to highlight the significant compensation of the disturbance effects when applying (DOB) into (PD) controller.

Fig. 3. Vibratory disturbances at the wheels and joints of the mobile manipulator.

Fig. 4. Trajectory tracking errors of X-axis and Y-axis with vibratory disturbances.
Fig. 5. Reference input torque at the wheels and joints of the mobile manipulator.

It is very obvious that the tracking performance of the proposed (PD-DOB) controller is significantly improved when compared with the case of the (PD) controller only. When vibratory disturbances are introduced into the system, the tracking errors of the (PD) controller are around $1.21 \, \text{mm}$ (for both X-axis and Y-axis) whereas the tracking errors for the (PD-DOB) controller are successfully suppressed to less than $1.34 \, \text{mm}$ (for both X-axis and Y-axis). Thus, the (PD-DOB) controller scheme is much more robust than the (PD) controller only in compensating the vibration effects.

The reference torque (Figure 5) are affected by vibratory disturbances, this is typically due to the effect of the disturbance observer controller (DOB) to guarantee correctly elimination of the imposed vibratory effects.

Fig. 6. Impact forces disturbances at the wheels and joints of the mobile manipulator.
It is obvious from the above results that the proposed (PD-DOB) controller is significantly improved when compared with the case of the (PD) controller only. The effect of the impact disturbances was considerably rejected by the proposed (PD-DOB) controller scheme. However, the tracking errors of the (PD) controller are around $5.62 \text{ cm}$ (X-axis) and $11.16 \text{ cm}$ (Y-axis) whereas the tracking errors for the (PD-DOB) controller are successfully suppressed to less than $3.12 \text{ mm}$ (X-axis) and $6.23 \text{ mm}$ (Y-axis). Notice that whenever there is an impact disturbances introduced into the wheels and joints of the mobile manipulator, the reference torque (Figure 8) take a sudden abrupt variation. This is typically due to the effect of the disturbance observer controller (DOB) in compensating the introduced impact effects.

Figures 5 and 8 show high frequency phenomena in the input torques. This is because the high frequency disturbance introduced into the system. To reduce this, properly designed filters in the loop of DOB controller can be utilized. Y. Huang and W. Messner [20] replaced the constant controller gain in the conventional disturbance observer with a first order filter. Compared with the conventional disturbance observer design, the proposed disturbance observer increase high frequency
disturbance attenuation without affecting the system stability. J. Teoh and C. Du [21] combined a phase stabilized feedback controller with a disturbance observer structure to achieve the rejection of disturbance. A high frequency bandpass filter is adopted in the disturbance observer in order to reject the high frequency disturbance. S. Komada and N. Machii [22] proposed an appropriate high-order disturbance observer system to improve system performance of hybrid position/force control of redundant manipulators in high and low frequency region. It has been shown that a proper selection of a coefficient of the disturbance observer is capable to improve robust stability without changing the basic performance of the system.

6. Conclusion

This paper describes a robust control strategy exploiting input-output decoupling controller when disturbances are imposed on the mobile manipulator. A classical proportional-derivative feedback structure plus an additional disturbance observer (PD-DOB) is proposed to compensate the external disturbances and uncertainties. The robustness of the proposed scheme has been demonstrated by simulation through a study on effect of the introduced vibratory excitation and impact force. The proposed controller is capable to reject a class of environmental disturbances and the tracking errors are effectively decreased by using the disturbance observer controller. However, further experiments are needed to investigate the maximum potentials of the scheme when other tasks, parameters or operating and loading conditions are considered. The practical issues related to the physical mobile manipulator should also be investigated.

References