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Robust stabilization under linear fractional parametric uncertainties of two-dimensional systems with Roesser models

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Abstract: This paper deals with the problem of robust stabilization for uncertain two-dimensional systems described by the Roesser model. The uncertainty under consideration has linear fractional form. Sufficient conditions for robust stability and robust stabilization are obtained. Moreover, the results generalize the works on uncertain 2-D systems with norm-bounded parametric uncertainties. A robust state feedback control law can be constructed based on solving a strict linear matrix inequality (LMI). Numerical examples are provided to demonstrate the applicability of the proposed methodology.
Keywords: 2-D discrete-time systems, stability, Linear Matrix Inequality (LMI), robust stability, robust stabilization, linear fractional parametric uncertainties.

1 Introduction

In recent years, there has been a growing interest in the study of two-dimensional (2-D) systems since these systems play important roles in describing systems in image data processing, water stream heating, thermal processes, gas absorption, etc [10]. The stability problem for 2-D systems has been studied in [1, 2], using 2-D Lyapunov equations. These results have been extended to the stabilization problem for 2-D systems [3, 4, 13].

Since modeling uncertainties are often the main source of instability of control systems, the problems of robust stability analysis and robust controller design for uncertain 2-D discrete systems have also received much attention: For example, the robust stabilization problem for 2-D systems has been addressed in [4, 13, 14], using approaches based on solving sets

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of LMIs. A parallel approach has been applied for the specific case of repetitive processes [6].

This paper is concerned with the problem of robust stabilization for uncertain 2-D discrete-time systems (An earlier version was presented in [8]. The class of 2-D discrete systems under consideration here is described by the Roesser state space model under linear fractional uncertainty form. The purpose is to design a full state feedback controller such that the resulting closed-loop system is asymptotically stable for all admissible uncertainties. A sufficient condition for the solvability of this problem is obtained and an LMI approach is developed, following the approach proposed for a different kind of systems by the authors [9]. Numerical examples are provided to demonstrate the application of the proposed method.

2 Problem formulation and Preliminaries

Consider the 2-D system (Σ) described by the following Roesser model:

$$(\Sigma) : \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \widetilde{A} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} + \widetilde{B}u(i,j), \tag{1}$$

where $x^h(i,j) \in \mathbb{R}^{n_1}$ is the horizontal state vector, $x^v(i,j) \in \mathbb{R}^{n_2}$ is the vertical state vector, $u(i,j) \in \mathbb{R}^m$ is the control input, and the time-invariant matrices $\widetilde{A}$ and $\widetilde{B}$ represent the system dynamics, maybe affected by uncertainties.
The following Assumption is imposed on the uncertainties:

Assumption 2.1 *The uncertainty in the system model (1) can be described using the following Linear Fractional Parametric model:*

$$\begin{bmatrix} \widetilde{A} & \widetilde{B} \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix} + H\Delta \begin{bmatrix} N_1 & N_2 \end{bmatrix} \tag{2}$$

$$\Delta(\xi) = [I - F(\xi)J]^{-1} F(\xi) \tag{3}$$

$$I - JJ^T > 0, \tag{4}$$

where A, B, H, N_1, N_2 and J are known constant matrices with appropriate dimensions. The uncertain matrix $F(\xi) \in \mathbb{R}^{l \times j}$ satisfies

$$F(\xi)F^T(\xi) \leq I, \tag{5}$$

where $\xi \in \Omega$, with Ω being a given compact set in $\mathbb{R}$.

Definition 2.1 *The class of parametric uncertainties Δ satisfying (2)-(5) is said to be admissible.*

Remark. The linear fractional form of parametric uncertainties has already been extensively investigated in 1-D robust control setting [7, 12].
Remark. Condition (4) guarantees that $I - FJ$ is invertible for all F satisfying (5).

Remark. The class of parametric uncertainties has been selected because it is very general, and includes other classes of uncertainties studied in the literature. For example, it is easy to see that the parametric uncertainties of linear fractional form reduce to norm bounded parametric uncertainties when $J = 0$, so the results can be easily particularized for this kind of uncertainties.

The unforced 2-D linear discrete-time system of (2)-(4) without uncertainty (i.e., nominal) is given by

$$(\Sigma_N) : \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = A \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}. \tag{6}$$

Throughout this paper, we shall adapt the following concept of asymptotic stability.

Definition 2.2 *[10]: The 2-D linear discrete-time system (Σ_N) is said to be asymptotically stable if*

$$\lim_{i,j\to\infty} \| x(i,j) \| = 0$$

under boundary conditions such that $\sup_j \| x^h(0,j) \| < \infty$ *and* $\sup_j \| x^v(i,0) \| < \infty$ *where* $x(i,j) = [x^h(i,j)^T, \ x^v(i,j)^T]^T$ *and* $\| x(i,j) \|$ *is the Euclidean norm.*

Now, we can cite the following result for the stability of 2-D systems:

Lemma 2.1 *[10] The 2-D linear discrete-time system (Σ_N) is asymptotically stable if there exists a block-diagonal matrix $P = diag(P_h, P_v) > 0$ with $P_h \in \mathbb{R}^{n_1 \times n_1}$ and $P_v \in \mathbb{R}^{n_2 \times n_2}$ such that*

$$A^T P A - P < 0. \tag{7}$$

The following result is useful for the synthesis problem.

Lemma 2.2 *The 2-D system (Σ_N) is asymptotically stable if there exist a block-diagonal matrix $Q = diag(Q_h, Q_v) > 0$ with $Q_h \in \mathbb{R}^{n_1 \times n_1}$ and $Q_v \in \mathbb{R}^{n_2 \times n_2}$ such that*

$$\begin{bmatrix} -Q & QA^T \\ AQ & -Q \end{bmatrix} < 0. \tag{8}$$

Proof: By pre-multiplying (8) by $diag(Q^{-1}, Q^{-1})$ and post-multiplying the result by $diag(Q^{-1}, Q^{-1})$, one has

$$\begin{bmatrix} -Q^{-1} & A^T Q^{-1} \\ Q^{-1} A & -Q^{-1} \end{bmatrix} < 0. \tag{9}$$

Let $Q^{-1} = P$; then, by the Schur complement equivalence, (9) leads to (7). Based on Lemma 2.1, (Σ_N) is asymptotically stable.

■

With the support of Lemmas 2.1 and 2.2, one can easily obtain the following result:

Lemma 2.3 *The 2-D system (Σ_N) is asymptotically stable if there exists a block-diagonal matrix $Q = diag(Q_h, Q_v) > 0$ with $Q_h \in \mathbb{R}^{n_1 \times n_1}$ and $Q_v \in \mathbb{R}^{n_2 \times n_2}$, such that*

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^TA^T & \frac{1}{2}V+V^T-Q \\ AV & -Q & -AV \\ \frac{1}{2}V+V^T-Q & -V^TA^T & -V-V^T \end{bmatrix} < 0. \tag{10}$$

In order to prove Lemma 2.3 the following result given in [5] will be used:

Lemma 2.4 *[11]: Given a real symmetric matrix Ψ and two real matrices M and R, the following LMI problem is solvable in the decision variable X*

$$\Psi + M^TX^TR + R^TXM < 0,$$

if and only if

$$\mathcal{N}_M^T \Psi \mathcal{N}_M < 0, \quad \mathcal{N}_R^T \Psi \mathcal{N}_R < 0,$$

where $\mathcal{N}_M$ and $\mathcal{N}_R$ are matrices whose columns form the bases of the right null space of M and R, respectively.

Proof of Lemma 2.3: Based on Lemma 2.2, it is only necessary to show that the feasibility of (10) for decision variables Q and V is equivalent to the feasibility of (8) for decision variable Q. Rewriting (10) as

$$\begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} + \begin{bmatrix} \frac{1}{2}I \\ -A \\ -I \end{bmatrix} V \begin{bmatrix} -I & 0 & I \end{bmatrix} +$$

$$\begin{bmatrix} -I \\ 0 \\ I \end{bmatrix} V^T \begin{bmatrix} \frac{1}{2}I & -A^T & -I \end{bmatrix} < 0. \tag{11}$$

If $M = \begin{bmatrix} \frac{1}{2}I & -A^T & -I \end{bmatrix}$ and $R = \begin{bmatrix} -I & 0 & I \end{bmatrix}$, explicit null space bases calculations yields

$$\mathcal{N}_M = \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A^T \end{bmatrix}, \quad \mathcal{N}_R = \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}. \tag{12}$$

Then, we have

$$\begin{aligned} \mathcal{N}_M^T \Psi \mathcal{N}_M &= \begin{bmatrix} I & 0 & \frac{1}{2}I \\ 0 & I & -A \end{bmatrix} \begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A^T \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2}Q & 0 & -Q \\ AQ & -Q & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ \frac{1}{2}I & -A^T \end{bmatrix} \\ &= \begin{bmatrix} -Q & QA^T \\ AQ & -Q \end{bmatrix} \end{aligned}$$

and

$$\mathcal{N}_R^T \Psi \mathcal{N}_R = \begin{bmatrix} I & 0 & I \\ 0 & I & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & -Q \\ 0 & -Q & 0 \\ -Q & 0 & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -Q & 0 & -Q \\ 0 & -Q & 0 \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & I \\ I & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -2Q & 0 \\ 0 & -Q \end{bmatrix}$$

which, by the projection Lemma 2.4, implies that inequality (10) is feasible in variable Q if and only if

$$\begin{bmatrix} -Q & QA^T \\ AQ & -Q \end{bmatrix} < 0, \tag{13}$$

and

$$\begin{bmatrix} -2Q & 0 \\ 0 & -Q \end{bmatrix} < 0. \tag{14}$$

Suppose that there exists a matrix Q such that (8) holds, that is inequality (13) holds, which implies

$$-Q < 0. \tag{15}$$

Inequality (15) implies that (14) holds. The desired result is obtained by using Lemma 2.2.

■

Remark 2. With the introduction of a new matrix V, we obtain a linear matrix inequality in which the Lyapunov matrix Q is not involved in any product with the dynamic matrix A. This feature enables one to derive a new robust stability condition which is less conservative due to the extra degrees of freedom (see the numerical example). It is noted that the matrix V introduced is not even constrained to be symmetric.

Motivated by the foregoing results, we introduce the following definition for the unforced system:

$$(\Sigma_a) : \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = \widetilde{A} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{16}$$

Definition 2.3 *System (Σ_a) is said to be robustly stable if, for any admissible uncertainty satisfying (2)-(5), any one of the following conditions is satisfied:*
a) There exists a block-diagonal matrix $\overline{P} = diag(\overline{P_h}, \overline{P_v}) > 0$ such that

$$\begin{bmatrix} -\overline{P} & \overline{P}\widetilde{A}^T \\ \widetilde{A}\overline{P} & -\overline{P} \end{bmatrix} < 0. \tag{17}$$

b) There exist matrices $\overline{P} = diag(\overline{P_h}, \overline{P_v}) > 0$ and V such that

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^T\widetilde{A}^T & \frac{1}{2}V + V^T - \overline{P} \\ \widetilde{A}V & -\overline{P} & -\widetilde{A}V \\ \frac{1}{2}V^T + V - \overline{P} & -V^T\widetilde{A}^T & -V - V^T \end{bmatrix} < 0. \tag{18}$$

Then, the objectives of this note are:
(i) to obtain conditions of the robust stability for (Σ_a).
(ii) to design a state feedback controller such that the closed-loop 2-D discrete-time system is robustly stable.

3 Robust stability

In this section, we give sufficient conditions for (Σ_a) to be robustly stable. To this end, the following lemma is needed.

Lemma 3.1 *[12]: Suppose that Δ is given by (3)-(5). Given matrices $M = M^T$, S and N of appropriate dimensions, the inequality*

$$M + S\Delta N + N^T \Delta^T S^T < 0$$

holds for all F such that $FF^T \leq I$, if and only if, for some $\delta > 0$

$$\begin{bmatrix} \delta M & S & \delta N^T \\ S^T & -I & J^T \\ \delta N & J & -I \end{bmatrix} < 0.$$

Theorem 3.1 *System (Σ_a) is robustly stable if any one of the following results is satisfied:*
a) There exists a block-diagonal matrix $P = diag(P_h, P_v) > 0$ such that

$$\begin{bmatrix} -P & PA^T & 0 & PN_1^T \\ AP & -P & H & 0 \\ 0 & H^T & -I & J^T \\ N_1 P & 0 & J & -I \end{bmatrix} < 0. \tag{19}$$

b) There exists a block-diagonal matrix $P = diag(P_h, P_v) > 0$ and a matrix V such that

$$\begin{bmatrix} -\frac{1}{2}(V + V^T) & V^T A^T & \frac{1}{2}V + V^T - P & 0 & V^T N_1^T \\ AV & -P & -AV & H & 0 \\ \frac{1}{2}V^T + V - P & -V^T A^T & -V - V^T & 0 & -V^T N_1^T \\ 0 & H^T & 0 & -I & J^T \\ N_1 V & 0 & -N_1 V & J & -I \end{bmatrix} < 0. \tag{20}$$

Proof: *a)* Suppose that (19) holds. Letting $P = \delta\hat{P}$ with $\delta > 0$ leads to

$$\begin{bmatrix} -\delta\hat{P} & \delta\hat{P}A^T & 0 & \delta\hat{P}N_1^T \\ -\delta A\hat{P} & -\delta\hat{P} & H & 0 \\ 0 & H^T & -I & J^T \\ \delta N_1 \hat{P} & 0 & J & -I \end{bmatrix} < 0,$$

which, using Lemma 3.1, implies that for any $\Delta(\xi)$ satisfying (3)-(5) the following inequality holds:

$$\begin{bmatrix} -\hat{P} & \hat{P}A^T \\ A\hat{P} & -\hat{P} \end{bmatrix} + \begin{bmatrix} 0 \\ H \end{bmatrix} \Delta \begin{bmatrix} N_1\hat{P} & 0 \end{bmatrix} + \begin{bmatrix} \hat{P}N_1^T \\ 0 \end{bmatrix} \Delta^T \begin{bmatrix} 0 & H^T \end{bmatrix}.$$

Then, we have

$$\begin{bmatrix} -\hat{P} & \hat{P}\widetilde{A}^T \\ \widetilde{A}\hat{P} & -\hat{P} \end{bmatrix} < 0.$$

Thus, the result follows by the part *a)* in Definition 2.1.
The proof of *b)* is similar to the one of part *a)*, so it is omitted.

■

In the case when $J = 0$, the parametric uncertainties of linear fractional form reduce to norm bounded parametric uncertainties:

$$\begin{bmatrix} \widetilde{A} & \widetilde{B} \end{bmatrix} = \begin{bmatrix} A & B \end{bmatrix} + HF \begin{bmatrix} N_1 & N_2 \end{bmatrix} \tag{21}$$

with $F(\xi)F^T(\xi) \leq I$. Therefore, when $J = 0$ the results of Theorem 3.1 reduce to the following:

Corollary 3.1 *: The system (Σ_a) is robustly stable for norm bounded parametric uncertainties if any one of the following results is satisfied:*
a) There exists a block-diagonal matrix $X = diag(X_h, X_v) > 0$, and a scalar $\epsilon > 0$ such that

$$\begin{bmatrix} -X & XA^T & XN_1^T \\ AX & -X + \epsilon HH^T & 0 \\ N_1X & 0 & -\epsilon I \end{bmatrix} < 0. \tag{22}$$

b) There exists a block-diagonal matrix $X = diag(X_h, X_v) > 0$, a matrix W and $\epsilon > 0$ such that

$$\begin{bmatrix} -\frac{1}{2}(V + V^T) & V^TA^T & \frac{1}{2}V + V^T - X & V^TN_1^T \\ AV & -X + \epsilon HH^T & -AV & 0 \\ \frac{1}{2}V^T + V - X & -V^TA^T & -V - V^T & -V^TN_1^T \\ N_1V & 0 & -N_1V & -\epsilon I \end{bmatrix} < 0. \tag{23}$$

Proof: *a)* When $J = 0$, inequalities (19) becomes

$$\begin{bmatrix} -P & PA^T & 0 & PN_1^T \\ AP & -P & H & 0 \\ 0 & H^T & -I & 0 \\ N_1P & 0 & 0 & -I \end{bmatrix} < 0,$$

which can be rearranged as

$$\begin{bmatrix} -P & PA^T & PN_1^T & 0 \\ AP & -P & 0 & H \\ N_1P & 0 & -I & 0 \\ 0 & H^T & 0 & -I \end{bmatrix} < 0.$$

It follows by the Schur complement equivalence that

$$\begin{bmatrix} -P & PA^T & PN_1^T \\ AP & -P + HH^T & 0 \\ N_1P & 0 & -I \end{bmatrix} < 0. \tag{24}$$

For some scalar $\epsilon > 0$, (24) can be written as

$$\begin{bmatrix} -(\epsilon P) & (\epsilon P)A^T & (\epsilon P)N_1^T \\ A(\epsilon P) & -(\epsilon P)+\epsilon HH^T & 0 \\ N_1(\epsilon P) & 0 & -\epsilon I \end{bmatrix} < 0. \tag{25}$$

Letting $X = \epsilon P$ leads to (21).
The proof of *b)* is omitted because the procedure is similar to that of part *a)*, so the proof is complete.

■

4 Robust stabilization

In this section, we give an LMI solution to the problem of control for (Σ). A state feedback controller

$$u(i,j) = K \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} \tag{26}$$

is used. Substituting (26) into (Σ) leads to the closed-loop system

$$(\Sigma_c): \begin{bmatrix} x^h(i+1,j) \\ x^v(i,j+1) \end{bmatrix} = [\widetilde{A} + \widetilde{B}K] \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}. \tag{27}$$

Recalling (2), one has that

$$\widetilde{A} + \widetilde{B}K = (A+BK) + H\Delta\,(N_1 + N_2K)\,. \tag{28}$$

Theorem 4.1 *: The closed-loop system (Σ_c) is robustly stable if any one of the following results is satisfied:*
a) There exists a block-diagonal matrix $P = diag(P_h, P_v) > 0$, and a matrix Z such that

$$\begin{bmatrix} -P & PA^T + Z^TB^T & 0 & PN_1^T + Z^TN_2^T \\ AP+BZ & -P & H & 0 \\ 0 & H^T & -I & J^T \\ N_1P + N_2Z & 0 & J & -I \end{bmatrix} < 0. \tag{29}$$

In this case, a robust stabilizing state feedback control law takes the form

$$u(i,j) = ZP^{-1} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}. \tag{30}$$

b) There exists a block-diagonal matrix $P = diag(P_h, P_v) > 0$, and matrices Z and V such that

$$\begin{bmatrix} -\frac{1}{2}(V+V^T) & V^TA^T & \frac{1}{2}V + V^T - P & 0 & V^TN_1^T + Z^TN_2^T \\ AV & -P & -AV & H & 0 \\ \frac{1}{2}V^T + V - P & -V^TA^T & -V - V^T & 0 & -V^TN_1^T - Z^TN_2^T \\ 0 & H^T & 0 & -I & J^T \\ N_1V + N_2Z & 0 & -N_1V - N_2Z & J & -I \end{bmatrix} < 0. \tag{31}$$

In this case, a robustly stabilizing state feedback control law takes the form

$$u(i,j) = ZV^{-1}\begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix} < 0. \tag{32}$$

Proof: The proof is trivial and is omitted.

■

5 Numerical Examples

First Example:
Firstly, consider the 2-D linear system with parametric uncertainty (Σ), based on the example in [13], defined by

$$A = \left[\begin{array}{cc|cc} 0.2 & 0.3 & 0.2 & -0.1 \\ 1 & 1 & 0 & 0.5 \\ \hline 0.8 & 0.2 & -1.3 & -0.1 \\ 0.2 & 0 & 1.3 & 0.1 \end{array}\right], B = \left[\begin{array}{cc} 0 & 1 \\ 1 & 0 \\ \hline 1 & 1 \\ 0 & 0 \end{array}\right],$$
$$H = \begin{bmatrix} 1 \\ 0.4 \\ 0.2 \\ 0.2 \end{bmatrix}, N_1 = \begin{bmatrix} 0.2 & 0.1 & 0.3 & 0.1 \end{bmatrix}, N_2 = \begin{bmatrix} 0.1 & 0.1 \end{bmatrix}, J = 0.5.$$

The uncertain matrix $\Delta(\xi)$ satisfies

$$\Delta(\xi) = \tfrac{F(\xi)}{1-0.5F(\xi)}, F(\xi) = \sin\xi.$$

It is easy to show that matrix

$$A_{22} = \begin{bmatrix} -1.3 & -0.1 \\ 1.3 & 0.1 \end{bmatrix}$$

contains an eigenvalue outside the unit circle given by -1.2. Therefore, the nominal unforced system is not asymptotically stable. The purpose of this example is to design a full state-feedback controller such that the closed-loop system is asymptotically stable for all admissible uncertainties. Using the Matlab LMI control Toolbox to solve the LMI (26), the following solutions can be obtained:

$$P = \begin{bmatrix} 23.0372 & -6.2133 & 0 & 0 \\ -6.2133 & 7.7667 & 0 & 0 \\ 0 & 0 & 3.6573 & -8.2847 \\ 0 & 0 & -8.2847 & 25.6795 \end{bmatrix},$$
$$Z = \begin{bmatrix} -17.2127 & -1.2540 & 4.0142 & -12.6354 \\ -2.2677 & 4.5835 & -1.5621 & 6.9432 \end{bmatrix}.$$

Thus, by Theorem 4.1 *a)*, a stabilizing state feedback is

$$u(i,j) = \begin{bmatrix} -1.0083 & -0.9681 & -0.0632 & -0.5124 \\ 0.0774 & 0.6521 & 0.6885 & 0.4925 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}.$$

The response of $x_1^v(i,j)$ of the open-loop system is shown in Figure 1, which shows that the open-loop system is unstable.

The responses of the closed-loop system of $x_1^v(i,j)$ for $F = 1$ and $x_2^v(i,j)$ for $F = -1$ are shown in Figures 2 and 3, respectively. The other state responses are similar, and hence, omitted. The simulation results show that the closed-loop system is asymptotically stable.

Second Example:

If we replace H in the first example with $H = \begin{bmatrix} 1.1 \\ 0.44 \\ 0.22 \\ 0.22 \end{bmatrix}$, the LMI (29) in Theorem 3.1 is unfeasible. However, the LMI (31) is feasible. The solutions of (31) are:

$$P = \begin{bmatrix} 90.2823 & -6.4914 & 0 & 0 \\ -6.4914 & 19.5367 & 0 & 0 \\ 0 & 0 & 9.9509 & -19.3054 \\ 0 & 0 & -19.3054 & 66.1499 \end{bmatrix},$$

$$Z = \begin{bmatrix} 1.3620 & -4.8679 & -2.1483 & 3.5233 \\ -4.0377 & 6.2192 & -8.8911 & 27.1153 \end{bmatrix},$$

$$V = \begin{bmatrix} 45.2988 & 0.2042 & 1.7712 & -0.3166 \\ -6.1857 & 18.1446 & -1.0808 & -2.5138 \\ -4.7135 & -3.8116 & 8.3859 & -7.6616 \\ -0.9337 & 1.0033 & -10.8358 & 36.3788 \end{bmatrix},$$

and the state feedback is:

$$u(i,j) = \begin{bmatrix} -1.0204 & -0.5261 & -1.5617 & -0.5181 \\ -1.0175 & -0.5293 & -1.5627 & -0.5188 \end{bmatrix} \begin{bmatrix} x^h(i,j) \\ x^v(i,j) \end{bmatrix}.$$

Thus, this second example shows that the introduction of the slack variables V give less conservative results, as there is a solution using Theorem 3.1 *b)* that is not feasible using *a)*.

6 Conclusion

This paper has investigated the robust stabilization problem for a class of 2-D discrete-time systems, described by the Roesser model in state space. The uncertainty under consideration is of linear fractional form. Sufficient conditions for robust stability and robust stabilization are obtained via the LMI approach. Two numerical examples are given to demonstrate the application of the proposed method.

It is important to notice that the proposed methodology can be applied to other kinds of 2-D systems: systems with delays, repetitive systems, etc.

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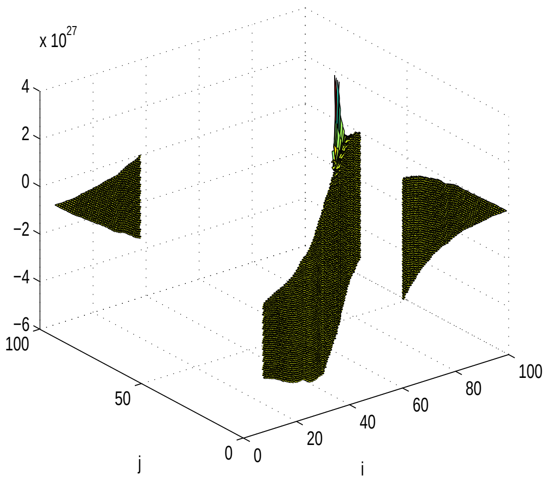
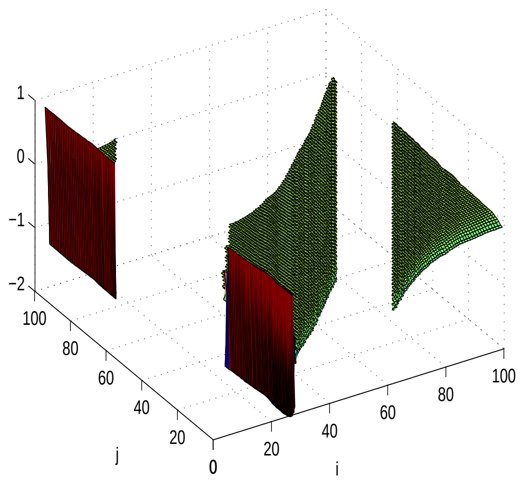


Figure 1. Open-loop response of $x_1^v(i,j)$.



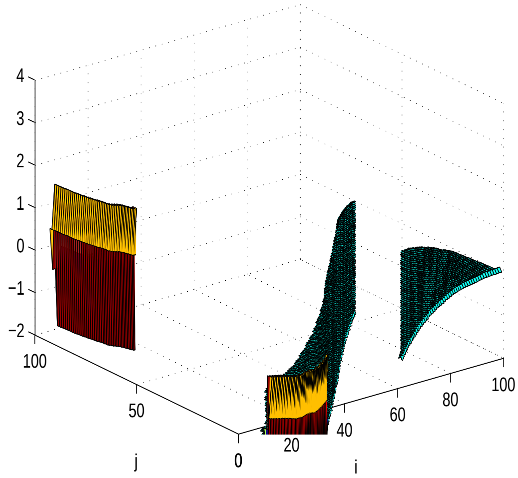


Figure 3. Closed-loop response of $x_2^v(i,j)$ for $F(\xi) = -1$.