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# Adaptive Observer for MIMO nonlinear systems: Realtime Implementation for an induction Motor

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**Abstract.** In this paper, we propose a global exponential adaptive observer for a class of uniformly observable nonlinear systems. This observer, based on techniques of high gain, is applied to jointly estimate missing states (rotor flux components) and unknown constant parameters incorporating in their expressions different electric parameters of an asynchronous motor. Finally, the feasibility of the proposed methodology is experimentally validated with a real-time implementation. The simulation results for induction motor are included to illustrate the effectivness of our design scheme.

Keywords: Non Linear System, High gain observer, induction motor drive

# **1** Introduction

Motors are electromagnetic devices used to convert electrical energy into useful mechanical work. There are two major classifications of ac motors. The first is induction motors that are electrically connected to the ac power source. Through electromagnetic coupling, the rotor and the stator fields interact, creating rotation without any other power source. The second is synchronous motors that have fixed stator windings that are electrically connected to the ac supply with a separate source of excitation connected to a field winding on the rotating shaft. Magnetic flux links the two windings when the motor is operating at synchronous speed. One major advantage of a induction motor over synchronous one is its ability to start and accelerate to

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steady-state running condition simply by applying ac power to the fixed stator windings of the motor [8].

Many works have been proposed some solutions of the problem of joint estimation of missing states and constant parameters in nonlinear state space systems with adaptive observers. However in most cases, adaptive observer have been proposed for a class of nonlinear systems which can be linearized with a change of coordinates up to output injection [1]. These methods assume the existence of some Lyapunov function satisfying particular condition [5] [9] [10]. In a recent work [6], an approach has been proposed for the synthesis of an adaptive observer for the time varying linear systems. First, an observer with constant gain of adaptation was been developed [6], then, in order to obtain a robust estimation of parameters, an observer with adjustable parameters has been suggested [4]. However, a difficulty in the synthesis of these observers resides in the choice of the observer for a class of uniformly observable nonlinear systems [2]. In this paper, we propose to generalize this synthesis in the purpose to elaborate some adaptive observers to single gain.

This paper is organized as follows. Section 2, presents the class of nonlinear MIMO systems. Section 3 interested to the design of the adaptive observer. Computer simulation and experimental results are illustrated in section 4. Section 5 gives a conclusion on the main works developed in this paper.

# **2** Problem Formulation

In this paper, we consider a class of Multi Inputs Multi Outputs (MIMO) nonlinear system whose dynamic can be expressed in the following form:

$$\begin{cases} \dot{x} = F(s(t))x(t) + G(s(t), u, x) + \Psi(s(t), x)\rho \\ y = \bar{C}x \end{cases}$$
(1)

The state  $x = \begin{pmatrix} x^1 & x^2 & \dots & x^q \end{pmatrix}^T \in IR^n$  with  $x^k \in IR^{n_k}$ ,  $k = 1, \dots, q$  et  $p = n_0$ 

$$\geq n_1 \geq n_2 \geq \ldots \geq n_q$$
 with  $\sum_{k=1}^q n_k = n$  et  $y \in IR^p$ ;

the input  $u \in U$  the set of bounded absolutely continuous functions with bounded derivatives from  $IR^+$  into U a compact subset of  $IR^s$ ; Let's denote :

#### Adaptive Observer for MIMO nonlinear systems - S. Aloui et al. 604

$$F(s(t)) = \begin{bmatrix} 0 & F_{1}(s(t)) & 0 & \cdots & 0 \\ 0 & 0 & F_{2}(s(t)) & \ddots & \vdots \\ \vdots & & \ddots & 0 \\ 0 & 0 & 0 & F_{q-1}(s(t)) \\ 0 & 0 & & 0 \end{bmatrix}, \quad G(s, u, x) = \begin{bmatrix} G^{1}(u, x) \\ G^{2}(u, x^{1}, x^{2}) \\ \vdots \\ G^{q}(u, x) \end{bmatrix},$$
$$\rho = \begin{pmatrix} \rho_{1} \\ \rho_{2} \\ \vdots \\ \rho_{m} \end{pmatrix}, \Psi(u, x) = \begin{bmatrix} \Psi_{1}^{T}(u, x) \\ \Psi_{2}^{T}(u, x) \\ \vdots \\ \Psi_{m}^{T}(u, x) \end{bmatrix}, \quad \overline{C} = \begin{bmatrix} I_{n_{1}} & 0_{n_{1} \times n_{2}} & 0_{n_{1} \times n_{3}} & \cdots & 0_{n_{1} \times n_{q}} \end{bmatrix}$$

 $\rho \in IR^{m}$  is the vector of unknown constant parameters,  $\rho_{i} \in IR$ , i = 1...m.

Our objective consists in making a study of performances, for an adaptive observer for the system (1). Finally, the feasibility of the proposed methodology is experimentally validated with a real-time implementation of induction motor in order to estimate rotor flux components and the unknown parameters of the induction motor.

# 3 Observer Design

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The adaptive observer synthesis requires at the time of their analysis a change of coordinates that will put the system (1) under a necessary particular form to the development of observers [2], [3]. In this section we limit ourselves to an observer's representation in the original coordinates.

While referring to works developed in the reference [2], the system (1) admits a global exponential adaptive observer. Her gain of observation ( $\theta$ ) is deduced of high gain techniques.

Before giving our candidate observer, we introduce the following notations. An adaptive observer for system (1) is:

$$\begin{cases} \dot{\hat{x}}(t) = F(s(t))\hat{x} + G(s(t), u, \hat{x}) + \Psi(s(t), \hat{x})\hat{\rho}(t) \\ -\Lambda^{+}(u, \hat{x})\Delta_{\theta}^{-1} \left(\theta S^{-1} + \Upsilon(t)\Gamma\Upsilon^{T}(t)\right)C^{T}(\overline{C}\hat{x} - y) \\ \dot{\hat{\rho}}(t) = -\theta \ \Omega_{\theta}^{-1}\Gamma\Upsilon^{T}(t)C^{T}(\overline{C}\hat{x} - y) \end{cases}$$
(2)

Where C and A are two matrixes given respectively by:

$$C = \begin{bmatrix} I_{n_1}, 0_{n_1}, ..., 0_{n_1} \end{bmatrix}$$
(3)

$$A = \begin{bmatrix} 0 & I_{n_1} & 0 & \cdots & 0 \\ \vdots & 0 & I_{n_1} & & \\ & \ddots & \ddots & \\ 0 & & 0 & I_{n_1} \\ 0 & \cdots & 0 & 0 \end{bmatrix}$$
 is a square matrix  $n_{1q} \times n_{1q}$ 

Let  $\Delta_{\theta}$  be the block diagonal matrix defined by:

$$\Delta_{\theta} = diag\left[I_{n_1}, \frac{1}{\theta}I_{n_1}, \dots, \frac{1}{\theta^{q-1}}I_{n_1}\right]$$

where  $\theta > 0$  is a real number.

Let  $\Omega_{\theta}$  be the following  $m \times m$  diagonal matrix where the  $v_k, k = 1, \dots, m-1$  are positive integers which are chosen such that each term of the matrix:  $\Delta_{\theta} \Psi(u, x) \Omega_{\theta}^{-1}$  is a polynomial in  $1/\theta$ .

$$\Omega_{\theta} = diag \left[ 1, \frac{1}{\theta^{V_1}}, \dots, \frac{1}{\theta^{V_{m-1}}} \right]$$
$$\Lambda(s(t)) = diag \left[ I_{n_1}, F_1(s(t)), F_1(s(t))F_2(s(t)), \dots, \prod_{k=1}^{q-1} F_k(s(t)) \right]$$

Let S be the unique solution of the algebraic Lyapunov equation:

$$S + A^T S + SA - C^T C = 0$$

where C, A are given, respectively, by equation (3). S is symmetric definite positive and that  $S^{-1}C^{T}$  is in the form:

$$S^{-1}C^T = \begin{bmatrix} C_q^1 I_{n_1} & C_q^2 I_{n_1} & \dots & C_q^q I_{n_1} \end{bmatrix}^T$$

Let  $\Upsilon(t)$  be a  $n_{1q} \times m$  matrix satisfying the following Ordinary Differential Equation:

$$\dot{\Upsilon}(t) = \theta \left( (A - S^{-1}C^T C) \Upsilon(t) + \Delta_{\theta} \Lambda(s(t)) \Psi(s(t)) \Omega_{\theta}^{-1} \right)$$

 $\Gamma$  is a matrix square constant inversible  $m \times m$ .

# 4 Application of the observer

In this paragraph, we apply the adaptive observer for the joint estimation of rotor flux and some parameters of the induction motor from the measure of stator currents and voltage components and the speed. Adaptive Observer for MIMO nonlinear systems - S. Aloui et al. 606

#### 4.1 Mathematical model for induction motor

Models of dynamical systems are used today in many technical and no technical fields, especially for simulation purposes. Such models are suitable for

- Getting a better insight into the system,
- Investigating and predicting special situations,
- Synthesizing feedforward and feedback systems.

To show the effectiveness of the proposed design scheme, its application to induction motor is provided. Induction motor has been employed in a wide range of industrial applications due to its reliability and low cost. However, since it exhibits highly nonlinear dynamics and its rotor variables are not usually measurable, it is essential to design a non linear observer that can estimate unmeasurable states. Induction motor is represented by fifth order nonlinear differential equation as (6):

$$\begin{cases} \frac{di_{\alpha_s}}{dt} = k(\frac{1}{T_r} \quad \phi_{\alpha_r} + p \quad \Omega_m \quad \phi_{\beta_r}) - \gamma \quad i_{\alpha_s} + \frac{1}{\sigma \quad L_s} v_{\alpha_s} \\ \frac{di_{\beta_s}}{dt} = k(-p \quad \Omega_m \quad \phi_{\alpha_r} + \frac{1}{T_r} \quad \phi_{\beta_r}) - \gamma \quad i_{\beta_s} + \frac{1}{\sigma \quad L_s} v_{\beta_s} \\ \frac{d\phi_{\alpha_r}}{dt} = -\left(\frac{1}{T_r} \phi_{\alpha_r} + P \quad \Omega_m \phi_{\beta_r}\right) + \frac{M}{T_r} \quad i_{\alpha_s} \end{cases}$$

$$\begin{cases} \frac{d\phi_{\beta_r}}{dt} = -\left(-p\Omega_m \phi_{\alpha_r} + \frac{1}{T_r} \phi_{\beta_r}\right) + \frac{M}{T_r} \quad i_{\beta_s} \\ \frac{d\Omega_m}{dt} = -\frac{C_r}{J} + \frac{pM}{JL_s} \quad (\phi_{\alpha_r} \quad i_{\beta_s} - \phi_{\beta_r} \quad i_{\alpha_s}) \end{cases}$$

$$\end{cases}$$

$$(6)$$

with:  $T_r = \frac{L_r}{R_r}$ ,  $\sigma = 1 - \frac{M^2}{L_s L_r}$ ,  $k = \frac{M}{\sigma L_s L_r}$ ,  $\gamma = \frac{R_s}{\sigma - L_s} + \frac{R_r M^2}{\sigma - L_s - L_r^2}$ 

where  $i_{\alpha_s}$ ,  $i_{\beta_s}$  are stator currents;  $v_{\alpha_s}$ ,  $v_{\beta_s}$  are stator voltage;  $\phi_{\alpha_r}$ ,  $\phi_{\beta_r}$  rotor flux components;  $\Omega_m$  is speed motor; *J* is a rotor inertia and *p* is the number of pole pairs;  $R_s$ ,  $R_r$  stator and rotor resistances respectively and  $L_s$ ,  $L_r$  stator and rotor inductances respectively.

In order to apply the algorithm developed to the induction motor, first, we proceed to the transformation of the model (6) in an identical form to the nonlinear system class definite by (1). We suppose:  $\rho_1 = \frac{1}{\sigma - L_s}$ ,  $\rho_2 = \gamma$ . These parameters will be joint estimated with variables states represented by rotor flux  $\phi_{\alpha_r}$  et  $\phi_{\beta_r}$  while supposing that  $i_{\alpha_s}$ ,  $i_{\beta_s}$ ,  $v_{\alpha_s}$ ,  $v_{\beta_s}$  et  $\Omega_m$  are as measurable. The motor model (6) can be putten under the form of the system (1) as follows:

$$\begin{cases} \begin{pmatrix} \frac{di_{\alpha_{r}}}{dt} \\ \frac{di_{\beta_{r}}}{dt} \\ \frac{d\phi_{\alpha_{r}}}{dt} \\ \frac{d\phi_{\alpha_{r}}}{dt} \\ \frac{d\phi_{\beta_{r}}}{dt} \\$$

The estimation of parameters  $(\rho_1, \rho_2)$  and the states variables  $(\phi_{\alpha_r} \text{ et } \phi_{\beta_r})$  is done, with the help of an adaptive observer of the form (2).

### 4.2 Simulation Results

In this section, the effectiveness of the proposed algorithm for jointly estimate missing states (rotor flux components) and unknown constant parameters incorporating in their expressions different electric parameters of an asynchronous motor is verified by computer simulation using MATLAB SIMULINK software according to Fig. 1.

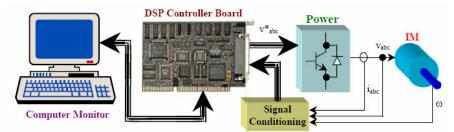


Fig.1. Induction motor estimation set-up

We opted for the following data: \*Nominal parameters of the induction motor are:

 $P_n = 1 \text{ Kw}, p = 2, L_s = 0.055 \text{ H}, M = 0.049 \text{ H}, J = 0.0035 \text{ Kg } m^2, R_s = 10\Omega, R_r = 1.69\Omega$ 

\* The initial conditions of this algorithm are chosen, as:

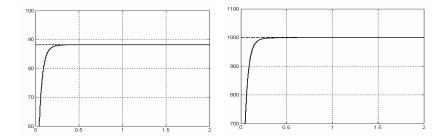
 $x(0) = \begin{bmatrix} 0.1 & 0.1 & 0.1 & 1 \end{bmatrix}^T$ ,  $\hat{x}(0) = \begin{bmatrix} 0.1 & 0.1 & 1 & 1 \end{bmatrix}^T$  and  $\hat{\rho}(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ 

\* The stator voltage admits the following expressions:

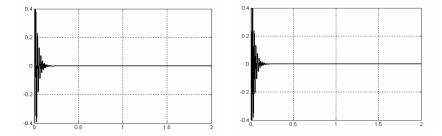
 $v_{\alpha_s} = 140(\cos(100\pi t))$  and  $v_{\beta_s} = 140(\sin(100\pi t))$ 

\* The adjustable parameters  $\Gamma$  and  $\theta$  used in the algorithm (2) are chosen, as:  $\Gamma = diag [1 \ 10]$  and  $\theta = 350$ .

In Fig.2, the real and estimated parameters  $\hat{\rho}_1(t)$ ,  $\hat{\rho}_2(t)$  are shown. While in Fig.3 the state estimation errors ( $\tilde{\phi}_{\alpha r} = \hat{\phi}_{\alpha r} - \phi_{\alpha r}$ , and  $\tilde{\phi}_{\beta r} = \hat{\phi}_{\beta r} - \phi_{\beta r}$ ) are given.



**Fig. 2.** Trajectories of the parameters  $\hat{\rho}_{1}(t)$ ,  $\hat{\rho}_{2}(t)$ Dashed line: real value. Solid line: estimed value



**Fig.3.** Estimated errors of states  $\tilde{\phi}_{\alpha r}$  and  $\tilde{\phi}_{\beta r}$ 

We can see from Fig.2 that the estimated parameters  $\hat{\rho}_1, \hat{\rho}_2$  converge to real parameters  $\rho_1, \rho_2$ .

Consequently, we can conclude, from results gotten in this application, that the quality of state estimation offered by the estimation algorithm as definite by the system (2) is good. The simulation shows that the observer is robust, in spite of the use of a non persistent excitation condition.

The purpose of the next section is to illustrate the feasibility of the proposed methodology through a practical application.

#### 4.3 Experimental results

4.3.1 Presentation of the plant:

In order to illustrate the performances of the adaptive observer (2), the proposed algorithm is applied to a laboratory plant consisting of a 1 kW squirrel cage motor of type LEYBOLD, a drake powder drake, inverters, a real time controller board of dSPACE DS1104 and interfaces which allow to measure the position, the angular speed, the currents, the voltages and the torque between the tested machine and the drake powder. The experimental setup, is depicted in Fig. 4.

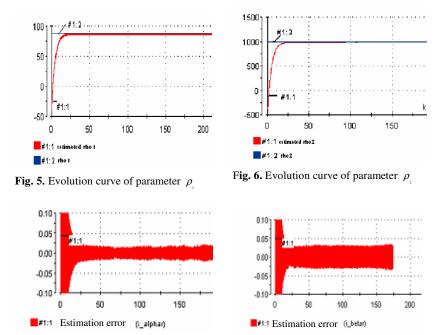


Fig. 4 View of the experimental setup

### 4.3.2 Experimental Results

curves of unknown parameters  $\rho_1$ ,  $\rho_2$  and those of state estimation errors are gotten for a rated speed 1500 tr / mn; observation gain  $\theta = 400$  and a matrix  $\Gamma$  defined by  $\Gamma = diag [7 7]$ .

We present Fig.5 and 6 curves of unknown parameters  $\rho_1, \rho_2$ . We illustrate, Figs.7 and 8 the state estimation errors.



**Fig. 7.** State estimation errors of  $i_{\alpha r}$ ,

From Fig.5 and 6 we state that the observer converges toward the different values of parameters  $\rho_1$ ,  $\rho_2$ . The convergence is completed at the discrete time k = 25, which corresponds to t = 2.5s.

From tracings of Fig.7, we remark that the estimation error is lower to 4%. We can conclude, from the experimental results presented previously, that the quality of estimation gotten following the real time implementation of adaptive observer (2) is satisfactory.

# 5 Conclusion

In this paper, we presented an adaptive observer for a class of uniformly observable MIMO nonlinear systems. This observer, based on techniques of high gain is applied to jointly estimate missing states (rotor flux components) and unknown constant parameters  $\rho_i$ ,  $\rho_j$  incorporating in their expressions different electric parameters of an induction motor ( $L_r$ ,  $L_s$ , M,  $R_s$ ,  $R_r$ ). An experimental setup has been developed and was used to verify the analysis. The algorithm is fast and simple and may easily be

implemented in real-time. The simulation and experimental results show good performances of the proposed approach.

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