Modeling of magnetic hysteresis and calculation of field in magnetic medium

Abdelaziz Ladjimi¹,
Mohammed El Rachid Mekideche²

¹Laboratoire de génie électrique de Guelma (L.G.E.G).
aziz_ladjimi@yahoo.fr
²Laboratoire d’études et de modélisation en électrotechnique université de Jijel (L.A.M.E.L).
mek_moh@yahoo.fr

Abstract. This work is devoted to analysis of the magnetic field of an electromagnetic device, taking into account the magnetic hysteresis. The latter implies a modeling appropriate cycle hysteresis. The cycle is represented by a model of Jiles-Atherton scalar. A module resolution was developed using the finite element method in 2D. Simulations made with the computation code have allowed us to study the impact of the phenomenon of hysteresis on the quantities magnetic such as the magnetic induction field and the potential vector.

Keywords. Magnetic hysteresis, Jiles-Atherton, finite elements, magnetic field

1. Introduction

Magnetic materials with useful physical and technological properties are exploited in many technical applications. They are present, for example, in electrical engineering, in motors or electrical transformers. Massive cut in sheet metal where they channel the magnetic flux. They give a memory and in other applications, conditioned in fine layers, they are employed as a support of recording.

For the control of magnetic materials, research focus in recent years towards the development of models characterizing the hysteresis cycle which is the most notable properties in these materials, models for the quantification of energy losses in these materials during their performance, and new materials more efficient.

The hysteresis magnetic modeling opens the way with the behavior hysteretic implementation of magnetic materials in the numerical analysis of the nonlinear magnetic fields often met in the problems and the applications of engineering. Currently, several models are used such as the Preisach model, the Stoner-Wohlfarth model, the Jiles-Aterton Model... etc.
The magnetic losses modeling allows the prediction and quantification of these losses during the operation of electromagnetic devices such as electrical machines, and therefore opens the way for performance improved and efficiency of machines using materials performance characterized by losses low.

The objective of this manuscript is to propose the modeling of the magnetic hysteresis and its association with the finite element analysis to study electromagnetic devices taking into account the phenomenon of magnetic hysteresis.

2. Jiles -Atherton scalar Models:

The theory of Langevin for paramagnetic materials gives the magnetization resulting from the orientation of the magnetic moments according to the direction of the field but disturbed by thermal agitation:

\[ M_{an}(H) = M_s (\coth(H/a) - H/a) \quad \text{avec} \quad a = \frac{kT}{\mu_0 m} \]  

Where \( M_s \) represents the saturation magnetization, \( H \) is the field of excitation applied, \( K \) the Boltzmann constant, \( T \) the absolute temperature, \( \mu_0 \) the permeability of the vacuum and \( m \) the elementary moment.

For ferromagnetic materials, we consider that there is an internal field (the molecular field of Weiss). It is then necessary, to obtain the equation of the anhysteretic curve, to correct the equation (1) by replacing the field \( H \) by the effective field of excitation \( h \) given by:

\[ h = H + \alpha M \]  

We introduce then the concept of fixing energy of the walls which leads to the equation giving the irreversible component of magnetization [1]:

\[ \frac{dM_{irr}}{dH} = \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} \]  

Where \( \delta \) is worth +1 when \( H \) increases and -1 when \( H \) decreases and \( M_{irr} \) is the irreversible component of magnetization, finally by adding the reversible component of magnetization [1], [2], [3], and supposing that:

\[ M_{rev} = c (M_{an} - M_{irr}) \]  

Where \( M_{rev} \) is the reversible component of magnetization. Then, total magnetization satisfied the equation:
\[
\frac{dM}{dH} = (1 - c) \left( \frac{M_{an} - M_{irr}}{k\delta - \alpha(M_{an} - M_{irr})} \right) + c \frac{dM_{an}}{dH} \tag{5}
\]

Where \( C \) is a constant.

The application of such an algorithm for the determination of the hysteresis loops by the model of Jiles-Atherton supposes the knowledge of the different parameters \( M_s, k, \alpha, c, \delta \) and \( a \).

### 3. Hystereses and finite elements:

The electromagnetic problems are described by Maxwell equations. In general case, we have:

- The Faraday's law:
  \[
  \text{curl}(E) = -\frac{\partial}{\partial t} B \tag{6}
  \]

- The Ampere's law:
  \[
  \text{curl}(H) = J \tag{7}
  \]

- The Flux conservation law:
  \[
  \text{div}B = 0 \tag{8}
  \]

\( E \) is the electric field, \( B \) magnetic induction, \( H \) the magnetic field and \( J \) the current density of excitation. We must associate these equations with the law of behaviour of magnetic materials:

\[
B = \mu_0 H + \mu_0 M \tag{9}
\]

\( M \) is magnetic magnetization and \( \mu_0 \) the magnetic permeability of the vacuum. The electromagnetic equation to solve after some manipulations in the vector potential \( A \) is written:

\[
\sigma \frac{\partial}{\partial t} A + \text{curl} \left( \frac{1}{\mu_0} \text{curl}(A) \right) = J + \text{curl} M \tag{10}
\]

Where \( \sigma \) is electrical conductivity.

The solution of this equation by the finite element method in 2D passes by the writing of an algebraic metrics system.

\[
[R_i] [A_j] = [S_j] + [F_j] \tag{11}
\]

- \( R_{ij} \) metrics rigidity
- \( S_j \) The excitation due to the current
- \( A_j \) vector potential
- \( F_j \) The excitation due to magnetization
4. Resolution methods:

The method used for the resolution of the nonlinear problem is a method of the type point fixed [4], [5] and [6]. It consists in repeatedly solving the problem until the convergence of the solution by integrating the model of hysteresis. The major difficulty of this calculation is the choice of a factor of relieving of the field. The algorithm used for simulations is represented in figure 2.

4.1. Studied device:

The device that we have chosen as the application is represented by a cylindrical ferromagnetic with a length of 40 cm and 8 cm in diameter. This cylinder is surrounded by a coil inductor of length L = 50 cm traversed by a current density of excitement \( J = 3 \times 10^6 \) A/m². Taking into account the axisymmetric nature of the problem we can consider only the 1/4 of the system (Figure 1).

The numerical values of parameters are in table (1)

**Table (1).** Hysteresis material parameters

<table>
<thead>
<tr>
<th>( M_s(A/m) )</th>
<th>( K(A/m) )</th>
<th>( \alpha(A/m) )</th>
<th>( c )</th>
<th>( \alpha )</th>
<th>( H_c )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6 ( 10^6 )</td>
<td>1000</td>
<td>1000</td>
<td>0.1</td>
<td>0.001</td>
<td>870(A/m)</td>
</tr>
</tbody>
</table>

Fig1. Test problem
Start

k=1, i=1, given $\mathcal{E}$

Initialization of $M$ and $P$

Calculate of $A^i$

Calculate $B^i$ ($B^i = \text{curl} (A^i)$)

Calculate of $H^i$ from.

$$H^i = \frac{1}{\mu_0} B^i - M^{i-1}$$

Relaxation of $H$

$$H^i = H^{i-1} + \omega (H^i - H^{i-1})$$

Relaxation coefficient ($0 < \omega < 1$).

Calculate $M^i$ from.

Jiels-Atherton model

Calculate precision.

$$\tau = \frac{|H^i - H^{i-1}|}{|H^i|}$$

End

$\tau \leq \varepsilon$

Yes

$i=i+1$

$\tau > \varepsilon$

Yes

$\tau < \varepsilon_f$

$K=k+1$

$\tau \leq \varepsilon_f$

No

$\tau > \varepsilon_f$

No

Fig. 2. Iterative algorithm
5. Results of simulation:

The resolution of the equation (9) by using the finite element code (EMF) which we
developed makes it possible to determine the evolution of the hysteresis loops in the
elements of the grid of the ferromagnetic load. We presented in figure 4 the
hysteresis loops of the selected elements (Figure 3), and in (figure 5) we presented the
hysteresis loops of all the elements of grid (of the load).

Fig. 3. Selected Elements position

Fig. 4. Hysteresis loops selected elements
Fig. 5. Hysteresis loops in all elements

The computation code which we developed makes it possible to determine the evolution of magnetic induction and the magnetic field according to time, as shows it figure 6 and figure 7.

Fig. 6. The evolution of magnetic fields for the selected elements according to time
Figure 6 shows the temporal evolution of the field applied for the selected element, \( H(t) \) presents the same form as the imposed current (sinusoidal form). Figure 7 shows the temporal evolution of magnetic induction calculated for the elements chosen; we can see that the wave forms of \( B(t) \) non-sinusoidal, those can translate the delay introduced by hysteresis between the field and induction on the deformation of the wave.

Figure 8 shows the evolution of the magnetic potential vector on the axis (Or), according to \( z \).

We can see that the value of the vector potential \( A \) is maximum at the level of the inductor then decreases until being cancelled in extreme cases of the field of study.

The distribution of the magnetic potential vector is highly concentrated on the level of the load as figure 9 shows it.

---

**Fig. 7.** The evolution of magnetic induction in the selected elements according to time

**Fig. 8.** Evolution of the vector potential \( A \) along the axis (or)
Fig. 9. Distribution of the vector potential A

**Conclusion**

In this work, we presented the static model of Jiels-Atherton. We have proposed a program under environment matlab, the program allowing the resolution of the nonlinear magnetodynamic problem by the integration of the model of Jiels-Atherton in a code of field by the finite elements.

The procedure suggested is applied for the calculation of the magnetic field of a device of induction heating, the results obtained are magnetic character namely the forms of waves, $H(t)$, $B(t)$, and distribution of the vector potential as well as the hysteresis loops are satisfactory this proves then, the reliability of the model and the method of resolution.

**References:**